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The perceived impact of PBL program learning on shifting prospective teachers' beliefs: A case study in China

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Abstract

As an important approach to learning and teaching, project-based learning (PBL) addressed in new curriculum reform at the middle school level in China raises the need for change in teachers' beliefs, since teachers' beliefs are important factors that influence their acceptance, adaptation, and implementation of PBL. Using qualitative content analysis, this study examined 23 prospective teachers' (PTs) perceived beliefs shifts through participating in a systematic PBL program at a university. Data were collected from two open questionnaires for pre- and post-tests, respectively, which were about the beliefs about the nature of mathematics (BNM), beliefs about learning mathematics (BLM), and beliefs about teaching mathematics (BTM). The results revealed that PTs' beliefs about BNM, BLM, and BTM were significantly shifted from the instrumentalist view or the Platonist view to the problem-solving view and that their shifts in BNM positively influenced the shifts in BLM and BTM. The implication for teachers' professional development is that transforming teachers' beliefs through systematic training on the theory and practice of PBL is feasible before implementing pedagogical innovations featuring PBL on a large scale.

Keywords: project-based learning, prospective mathematics teachers, teacher training program, teachers' beliefs, shift in beliefs

INTRODUCTION

In recent years, middle schools in China have been promoting initiatives to innovate in pedagogy and improve educational quality, aiming to support students in developing such core competencies as problem-solving, communication and collaboration, critical questioning, creativity, and activity experiences while acquiring academic knowledge and skills (Cai & Jiang, 2017; China Ministry of Education, 2022; Wang et al., 2018; Yao & Guo, 2018). In this process, project-based learning (PBL) has been increasingly highlighted in middle school teaching, a trend that is visible in several newly issued official documents. In June 2019, the Central Committee of the Communist Party of China and the State Council of China issued Opinions on Deepening Education Teaching Reform and Comprehensively Improving the Quality of Compulsory Education in which it was explicitly proposed that schools at the compulsory education level (which includes middle school) should further optimize their instructional methods by guiding students to think proactively, question actively, and explore cooperatively through research-based, project-based, and cooperative learning. As the most educationally developed regions in China, Beijing, Shanghai, and Zhejiang have launched large-scale curriculum and teaching reform initiatives featuring PBL. For instance, in October 2020, the Shanghai Municipal Education Commission issued the three-year action plan for PBL in compulsory education in Shanghai (2020 to 2022), proposing PBL as a focal point for promoting changes in the city's instructional approaches to further enhance schools' vitality and develop students' core literacy. By implementing this plan, it is expected that approximately 100 experimental schools featuring PBL will be cultivated and a series of replaceable cases of PBL will be developed.

In this context, PBL in mathematics education has been formally introduced in official documents. China Ministry of Education recently issued the mathematics

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Contribution to the literature

- Rather than most research focusing on adopting PBL as a pedagogical approach in prospective teacher training and exploring its effects on prospective teachers' academic performance, and self-efficacy beliefs, this study focuses on incorporating the theory and practice of PBL in middle schools into prospective teacher training curricula to examine the effects of such curricular experiences on shifting prospective teachers' beliefs.
- This study verified the feasibility and effectiveness of incorporating theoretical and practical knowledge of PBL in middle schools in prospective teacher training courses to shift their beliefs, which provides references for upgrading the prospective teacher training curriculum.
- With a sample of prospective teachers in China, this study confirmed the mechanism that shifts in teachers' beliefs usually begin with a shift in their beliefs about the nature of mathematics.

curriculum standards for compulsory education (2022 edition) in which PBL was first explicitly proposed as a model for students in middle school to learn integration and practice, a parallel strand to numbers and algebra, graphics and geometry, probability and statistics, aimed at helping students integrate mathematics and other disciplines, develop problem-solving skills, and foster the ability to "observe the real world through mathematical eyes, reflect on the real world with mathematical thinking, and express the real world in mathematical language" (China Ministry of Education, 2022).

According to the literature (Blumenfeld et al., 1991; Chen & Yang, 2019; Krajcik et al., 2008; Markula & Aksela, 2022; Rivet & Krajcik, 2008; Rogers et al., 2011), PBL advocates learning in contextualized, authentic situations in which students are required to engage in group work and sustained inquiry, and to accomplish a product that develops content knowledge and critical thinking. Although PBL has become or is becoming, an important direction for curriculum reform, frontline teachers may encounter challenges when implementing it. These challenges lie in the requirement for teachers to master additional, previously unknown knowledge and skills (Han et al., 2015b), embrace a constructivist and student-centered philosophy and have a positive, receptive attitude towards this emerging pedagogical approach (Blumenfeld et al., 1991; Mentzer et al., 2017; Rogers et al., 2011; Tamim & Grant, 2013). Previous research has shown that there exists a potential gap between PBL as understood by teachers and that defined in the documents (Tamim & Grant, 2013), the latter of which can be interpreted as the expected and ideal PBL. Therefore, to support teachers in accurately implementing PBL, systematic training programs are required to help them learn the relevant theoretical knowledge, engage in the practice of curriculum design (Han et al., 2015b), and modify their beliefs about learning and teaching (Rogers et al., 2011). It is worth noting that teachers' beliefs are important factors that influence their teaching practices (Ball & Cohen, 1996; Cross, 2009; Lavy & Shriki, 2008; Philipp, 2007; Torff, 2005; Wilkins, 2008; Wilson & Cooney, 2002). Currently, regions such as Shanghai and Beijing are pioneering reform initiatives featuring PBL. Considering that these reform programs will be replicated in more regions in the future, training future teachers who can implement PBL is necessary. It is suggested that prospective teachers (PTs) learn about the implementation of PBL in their university coursework (Markula & Aksela, 2022). In this context, it becomes essential to supplement the PTs education curriculum with content on the theory and practice of PBL. The present study aims to explore the feasibility of these course experiences in shifting PTs' beliefs by incorporating theoretical and practical courses of PBL into the curriculum to provide a reference for reforming curriculum design for PTs in the context of the popularity of PBL in middle schools. Therefore, our research question is, as follows:

Does the introduction of PBL-related theoretical and practical courses in PTs training programs have any perceived impact on their beliefs about mathematics, mathematics learning, and mathematics teaching at the middle school level?

LITERATURE REVIEW & FRAMEWORK

Practice and Challenges of Project-Based Learning

Although the definition of PBL varies across studies, a consensus has been reached on the core concepts of authentic situations, sustained inquiry, collaborative learning, and product orientation that PBL promotes. This implies that students' growth in the PBL environment is multifaceted. International empirical studies have shown that by conducting PBL, in addition to significantly improving students' academic achievement (Balemen & Keskin, 2018; Bilgin et al., 2015; Chen & Yang, 2019; Han et al., 2015a; Harris et al., 2015; Karacalli & Korur, 2014), their information literacy (Chu et al., 2011), problem-solving competencies (Ferreira & Trudel, 2012; Mettas & Constantinou, 2008), critical thinking and question-posing competencies (Sasson et al., 2018), and communication competencies (Owens & Hite, 2022) can also be significantly enhanced. Furthermore, their motivation to learn (Duke et al., 2021), attitudes towards mathematics, science, and other subjects (Ferreira & Trudel, 2012; Tseng et al., 2013), and interest in learning and engagement in the classroom (Remijan, 2017; Wurdinger et al., 2007) are also enhanced. Several studies have even shown that PBL has a positive impact on academic achievement and nonintellectual factors of students at risk and in low-socioeconomic-status schools (Duke et al., 2021; Holmes & Hwang, 2016). PBL has yielded positive results in practice. However, this does not necessarily mean that teachers do not encounter various challenges in implementing it. According to existing studies, some of these challenges stem from external reasons, such as resources and support provided by the schools and the time available for its implementation (Lam et al., 2010; Viro et al., 2020). Other challenges can be explained in terms of teachers' subjective factors. It has been found that when implementing PBL, some teachers were concerned about the difficulties in helping students adapt to this novel approach and doubted its facilitative effect on teaching and learning, thus returning to the traditional approach of drill and practice after a period of experimentation. Others considered that the collaboration and creative thinking that PBL promotes are not normally covered in standardized tests, and this discrepancy between assessment and instruction caused them to lose interest in PBL and eventually abandon it (Rogers et al., 2011). From the perceived risks of teachers mentioned above, teachers consider that sustained inquiry is time-consuming, while traditional instruction is cost-effective. It can be concluded that the subjective causes of the ineffectiveness of implementing PBL are the teachers' failure to identify with PBL and the lack of change in their previous beliefs.

Mathematics Teachers' Beliefs

Beliefs have not been uniformly defined in previous studies. In a summary of earlier research, Cross (2009, p. 326) considered beliefs as "embodied conscious and unconscious ideas and thoughts about oneself, the world, and one's position in it, developed through membership in various social groups." According to the viewpoint proposed in the Teacher education and development study: Learning to teach mathematics (TEDS-M) by the International Association for the Evaluation of Educational Achievement (IEA), beliefs and knowledge are two essential dimensions of teachers' necessary literacy (Blömeke & Kaiser, 2014; Kutaka et al., 2018). Teachers' beliefs impact their pedagogical decisions (Philipp, 2007) and are predictors of students' learning outcomes (Marshall et al., 2009).

In mathematics education, various studies agree on the classification of teachers' beliefs into three main dimensions: beliefs about the nature of mathematics (BNM), beliefs about learning mathematics (BLM), and beliefs about teaching mathematics (BTM) (Beswick, 2005; Cooney et al., 1998; Cross, 2009; Ernest, 1989; Vesga-Bravo et al., 2022; Xie & Cai, 2021). BNM refers to the idea of "what mathematics is", while BLM and BTM are about "how to learn mathematics" and "how to teach mathematics", respectively. According to Ernest (1989), BNM consists of three levels:

(1) the instrumentalist view in which people tend to believe that mathematics is a collection of facts, skills, procedures, rules, etc.,

(2) the Platonist view in which people believe mathematics is a preexisting body of knowledge to be found, and

(3) the problem-solving view in which people believe mathematics is in dynamic development and that its domain expands as people explore it.

These three distinct BNMs lead teachers to different orientations of BLM and BTM (Cross, 2009). Ernest (1989) argued that teachers with the instrumentalist view might ignore the connections among concepts in their teaching, teachers with the Platonist view may recognize the importance of the mathematical system and the connections among concepts, and teachers with the problem-solving view may emphasize students' autonomous exploration of knowledge.

Teachers' Beliefs and the Implementation of Project-Based Learning

Teachers' beliefs are the guidance supporting their teaching practices (Cross, 2009; Torff, 2005; Wilkins, 2008). As mentioned above, teachers encounter challenges in implementing PBL not only from external variables (e.g., schools' commitment of resources and time) but also from the consistency between their own held beliefs and the philosophy of teaching and learning promoted by PBL. Teachers' understanding and implementation of the philosophy promoted by PBL is an essential factor in students' knowledge understanding and skill development (Capraro et al., 2016), and only if teachers maintain a high level of pedagogical fidelity to this philosophy in the implementation of PBL can students truly benefit from it and achieve higher scores (Han et al., 2015b). Taking the beliefs about teaching as an example, in PBL, the teacher's perception of his or her role needs to shift from a lecturer or instructor of knowledge to a facilitator of learning (Han et al., 2015b), which requires teachers to shift their beliefs from instrumentalist or Platonist view to problem-solving view that embrace student-centeredness, autonomous exploration, etc. (Mentzer et al., 2017; Tamim & Grant, 2013). Only when teachers develop this well-grounded belief system and maintain a high level of fidelity in their practice can the ideas promoted by PBL truly be realized in the classroom teaching.

Course Experiences and Shifts in Prospective Teachers' Beliefs

Many countries are vigorously promoting student-centered, constructivist-oriented curriculum reform (Empson & Junk, 2004; Isikoglu et al., 2009; Rich, 2021), with China being no exception (China Ministry of Education, 2022; Wang, 2011). It has been suggested that the lesson plans that teachers develop are determined more by the beliefs they hold than by the ideas of curriculum reform, suggesting that teachers' beliefs are the foundation of success in curriculum reform (Vesga-Bravo et al., 2022). For PTs, beliefs are mainly derived from their own learning experiences and interactions with teachers (Bernack-Schüler et al., 2015; Jao, 2017; Lavy & Shriki, 2008; Thompson, 1992), and such beliefs may be limited (Swars et al., 2007).

The exposure of PTs to training programs during their college years can provide them with a plethora of vivid experiences in epistemic reflection that shape the academic environment that supports their shifting beliefs (Bernack-Schüler et al., 2015). Fortunately, research has confirmed that there is potential for PTs to shift their existing beliefs about learning in a mathematics methods course (Jao, 2017).

In the case of PBL, teachers' beliefs are the guiding force for their identification with PBL and the determinant of their commitment to implementing PBL (Rogers et al., 2011). According to the earlier analysis, the more teachers agree with the beliefs of the problem-solving view, the more receptive they might be to accepting and implementing PBL. In turn, it can be inferred that a promising solution for teachers to develop the beliefs of the problem-solving view might be to expose them to theoretical knowledge and hands-on involvement in PBL during their professional training as teachers at the university. Given that the experience of implementing PBL has brought conflict between teachers' deep-rooted beliefs about teaching and learning (Tamim & Grant, 2013), we consider such an inference to be meaningful and, therefore, worth exploring through research.

Research on teachers' beliefs in different cultures is necessary, as it may provide meaningful insights into each other (Engeln et al., 2013; Kutaka et al., 2018). For instance, by comparing teachers' beliefs about mathematics among twelve countries, Engeln et al.'s (2013) study found that the German mathematics classroom is highly teacher-centered since teachers are influenced by traditional teaching beliefs. Conversely, teachers' beliefs about teaching and learning in Eastern countries have shifted considerably in the evolving educational atmosphere, becoming increasingly student-centered. This cross-cultural comparative study of teachers' beliefs helps mathematics teachers and mathematics teaching researchers in different cultures to deepen their understanding of their mathematics education and to enhance their perceptions of the status of mathematics education in different countries through the lens of teachers' beliefs. However, not much research has been conducted on Chinese teachers' beliefs (Cai, 2004; Li et al., 2020; Yang et al., 2020). Research has shown that teachers' understanding and implementation of PBL affect the extent to which students benefit from this emerging approach and that if teachers do not implement PBL well, which implies a low instructional commitment to this approach, then it is of limited help to students and may even negatively impact their academic performance (Capraro et al., 2016; Erdogan et al., 2016; Han et al., 2015b; Markula & Aksela, 2022).

Moreover, most of the studies that have addressed both PBL and PTs have explored the effect of using PBL as an instructional approach on PTs' academic performance (Baran & Maskan, 2011; Bilgin et al., 2015; Mettas & Constantinou, 2008; Papastergiou, 2005), on enhancing self-efficacy beliefs towards teaching and decreasing anxiety about teaching (Bilgin et al., 2015; Novak & Wisdom, 2018), or on PTs' experiences in a PBL setting (Dag & Durdu, 2017). However, few studies have introduced the theoretical knowledge and essential practical skills in mathematics teacher training courses for the implementation of PBL in middle schools or have examined the effect of such attempts on their belief shifts.

Framework

As mentioned above, students' BNM is closely associated with their BLM and BTM. According to Cross's (2009) summary of earlier studies (Cobb & Steffe, 1983; Dionne, 1984; Ernest, 1989; Kuhs & Ball, 1986; Lindblom-Ylanne et al., 2006), the three dimensions of teachers' beliefs are interpreted as follows. The instrumentalist view is traditionalist, which focuses on teachers transmitting knowledge and students receiving it. The problem-solving view is constructivist, which emphasizes that learning should be student-centered, and students should be allowed to explore and create. The Platonist view is formalist, which is a belief that falls between the two previous beliefs, that mathematics is neither informed nor created and that mathematics is not a mere accumulation of discrete knowledge but rather a system of connections, whereby mathematics is discovered. Based on a systematic analysis of the relationships among BNM, BLM, and BTM, previous research constructed a matrix consisting of three dimensions and three levels as a framework for depicting teachers' beliefs (Beswick, 2005). To operationalize the dimensions for analytical purposes, we specified the existing framework by dividing BNM into two subdimensions of objective orientation and knowledge organization, BLM into two subdimensions of learning approach and teacher-student relationship,

Table 1. The framework for depicting teachers' beliefs

	BNM		BLM		BTM	
Level	Objective orientation	Knowledge organization	Learning approach	Teacher-student relationship	Teachers' role	Teaching format
Instrumentalist view	Knowledge & skills in textbooks	Focus on knowledge & skills	Passive reception of knowledge	Teacher-centered	Interpreter of knowledge	Closed lecture
Platonist view	Meaningful understanding	Focus on conceptual & cognitive structures	Active construction of understanding	Teacher-guided	Guides for learning	Semi-open & semi-closed
Problem-solving view	Integrated literacy	Catering to student exploration	Autonomous exploration of own interests	Learner-centered	Facilitator of learning	Open inquiry

and BTM into two subdimensions of teachers' role and teaching format (as shown in **Table 1**).

METHOD

Participants

23 participants, eight males, and 15 females, were enrolled in this study, all of whom were juniors from the Education Department of Beijing Normal University. Beijing Normal University is a comprehensive university dedicated to PTs preparation, ranking first among its counterparts in China, and one of its core missions is training exceptional teachers. These 23 undergraduates may not choose to teach as a career in the future. However, given that Beijing Normal University is a university with characteristics and strengths in teacher education and that past graduates are highly expected to pursue careers as teachers, either after undergraduate or graduate school, we consider these students to be PTs in this study, despite the possibility that they may choose other careers in the future.

We informed these PTs of our intention to conduct the study and promised them that the pre- and post-tests involved in this study were not related to course grades to encourage them to fill out the questionnaire voluntarily, carefully, and objectively. All participants were coded anonymously; for example, MYY01 represents a participant whose initials are MYY and whose number is 01.

Program Design

In light of the recent curriculum reform that has introduced PBL at the middle level (China Ministry of Education, 2022), it is necessary to strengthen PTs' training in the theoretical and practical knowledge of PBL, as they are expected to benefit from this knowledge and experience when they graduate from universities and enter K-12 schools in a few years. According to Friedrichsen et al. (2009), there are three main sources of teacher knowledge: teacher education programs, their own learning experiences, and teaching experience. Considering that the PTs in this study had no previous learning experiences or knowledge related to PBL and had no prior teaching experience, we designed a dedicated program in the teacher education course (i.e., theory and practice of instructional design in middle school mathematics) for them to acquire the related knowledge.

In the middle of the course schedule, five weeks are dedicated to introducing the theory and practice of PBL. Specifically, two hours per week for 10 hours, including four hours for the theoretical course and six hours for the practical course. In the theoretical course, the syllabus and teaching program was designed by the lecturer, and the concept, design principles, implementation principles, and typical cases of PBL were delivered successively. Based on the recommendations of previous studies (Bernack-Schüler et al., 2015; Papastergiou, 2005; Savery & Duffy, 1995), in the practical course, we emphasized providing PTs with opportunities for case design and epistemic reflection, encouraging them to build teams, select content for mathematics, design a preliminary case for a complete PBL, discuss and refine the case within the group, and engage adequately in other processes. At the end of the program, each group presented their designed case in class, commented on each other's cases, and exchanged the impressions gained from the design. The lecturer and teaching assistants conducted a comprehensive evaluation of each group's case design.

Questionnaire

According to Safrudiannur and Rott (2020), it is appropriate to adopt a qualitative research method to examine teachers' beliefs. In this study, an open-ended questionnaire was applied to collect data on PTs' beliefs. Following the methods of previous studies (Bernack-Schüler et al., 2015), to depict the shifts in PTs' beliefs, it is necessary to perform pre- and post-tests.

Three open-ended questions were designed for the pre-test. Question 1 asked, "what do you know about mathematics in middle school?", and question 2 asked, "what do you know about learning mathematics in middle school?" In answering these two questions, the participants were asked to first summarize four

Table 2. Examples of the perceived shifts in the three types of beliefs

Perceived shifts	BNM (Q1, Q3, & Q4)	BLM (Q2)	BTM (Q4)
(1) Instrumentalist view→(2) Platonist view			
(2) Platonist view→(3) Problem-solving view		Increased agreement with "interaction"	"Knowledge interpreter"→ "Learning facilitator"
(1) Instrumentalist view→(3) Problem-solving view	"Double-base"→ "Problem-solving"		

descriptive words (including but not limited to adjectives, metaphors, and examples, and then to express their answers in at least 300 words (in Chinese). In question 3, the students were asked to draw a "mathematical tree" and to explain the meaning of the drawing in a paragraph that described the imagery of mathematics in their minds.

Four open-ended questions were designed for the post-test of which questions 1, 2, and 3 were identical to the pre-test. In question 4, the participants were asked to complete a reflection on the program based on the following:

> "Dear friends, no matter what your future career plans are, you may aspire to become a teacher at this moment or in the future. After these five sessions of theoretical and practical experiences with PBL, have your perceptions of "mathematics" and "mathematics teaching" changed? If so, what do you think has changed? Please respond carefully based on your actual situation."

Three points need to be explained concerning the rationale of the above design. First, the breadth and depth of knowledge in mathematics vary considerably across school levels, and how mathematics is learned and taught also differs. To narrow the questions to avoid scattered responses, participants in this study were limited to responding only to the "middle school" level. The reason for choosing this level was that the cases involved in the course (i.e., theory and practice of instructional design in middle school mathematics) that these participants were studying were mainly from middle school mathematics. Second, the question "what do you know about teaching mathematics in middle school?" was not designed for the pre-test because PTs had no practical experience in teaching before the pre-test; however, during the practical course on PBL, they reflected on and evaluated the scientific and rational aspects of the cases from a pedagogical perspective and accumulated personal experience in designing mathematics instruction. Given this, only such a question (i.e., question 4) was designed in the post-test. Third, question 3 in the pre- and post-tests examined the mathematical imagery of the participants as a supplement to question 1 and therefore was only used as supporting information in the analysis for the results section.

Data Analysis

The data material consists of PTs' utterances in the pre- and post-tests, and students' portfolios looking for a supplement. The data were analyzed with methods inspired by qualitative content analysis (i.e., Mayring, 2004) but without the objectivist connotations related to them (Charmaz, 2006). Following the standards and procedures of qualitative research to ensure the validity and reliability of this study (Noble & Smith, 2015), we selected teachers' beliefs framework (**Table 1**) based on the research questions and then collected and analyzed data regarding our framework. Specifically, to examine the PTs' change in views of the BNM, BLM, and BTM, we followed three consecutive phases for data analysis. Firstly, we scanned all the PTs' utterances from the questionnaires through the lenses that concerned their perception of the image of BNM, BLM, and BTM. When scanning the PTs' utterances, we noticed that they could be classified into three levels of beliefs (the instrumentalist view, the Platonist view, and the problem-solving view) according to the framework (see **Table 1**), as shall be explained broadly in the results section.

Secondly, to validate our classification into the three aspects (the instrumentalist view, the Platonist view, and the problem-solving view) and the distinction between declarative and operatively oriented utterances, two university researchers (authors of this article) and one graduate student read the whole corpus of utterances and coded them according to our framework. We discussed the disagreement parts of the coding among us until we reached an agreement (Korstjens & Moser, 2018). In addition, we added the relevant evidence in the PTs' portfolios to support our conclusions.

Finally, from the above-analyzed data, we could gain a sense of the PTs' perceived image of the BNM, BLM, and BTM. We use the term 'perceived image' since it reflects the students' said beliefs regarding various aspects that concern the views of the BNM, BLM, and BTM, and does not necessarily reflect their practical abilities and skills. More detail about the coding and evidence of the examples of perceived shifts in BNM, BLM, and BTM, as well as its related questions in the questionnaire are shown in **Table 2**.

RESULTS

Shifts in Beliefs About the Nature of Mathematics

We found that the BNM held by PTs was not definitively attributable to a particular level but rather to elements of several levels simultaneously, which corroborates previous research (Beswick, 2012). By conducting systematic courses on the theory and practice of PBL, PTs' BNM shifted with an increased agreement and acceptance of the problem-solving view.

Objective orientation: From "double-base" to "problem-solving"

The comparison between the pre- and post-tests revealed that most of the PTs' BNM showed a shift from the instrumentalist view to the problem-solving view and considered that the objective orientation of mathematics should move from focusing on the "double-base" (i.e., basic knowledge and basic skills) to problem-solving.

For instance, participant MYY01 reflected on her previous instrumentalist view:

> My perception of "what mathematics is" has changed significantly before and after the course. My previous perception was simpler, thinking that mathematics is the sum of rational thinking and concepts. However, after experiencing the five courses and practices of PBL, I feel that the deeper meaning of mathematics is its occurrence and application, which means that mathematics comes from real life and is applied to life in which the application ability is very important.

Some participants had a deeper understanding of the connection between mathematics and life. For example,

> Mathematics does not exist in isolation but is deeply connected to daily life. I used to be deeply influenced by the traditional thinking that mathematics is only in the form of theorems, application problems, and so on. However, now I find that mathematics can also be derived from real life and then applied to real life (QJQ08).

> Mathematics is not only knowledge but also integration with life. Knowledge comes from life and eventually melts into life (XWJ06).

> Due to the influence of my major, my understanding of mathematics used to stay at a more theoretical level, but this course has shown me how mathematics practically permeates life, as evidenced by the PBL cases designed by our groups (CML17).

Some participants emphasized the contextual aspect of mathematical knowledge:

> My understanding of mathematics changed from a single view to a context-based view. Previously, my knowledge of mathematics was limited to the concepts in textbooks. After the introduction to the PBL course, I found that mathematics exists everywhere in life, whether it is algebra or geometry. Mathematics based on real-life situations gives a stronger sense of reality (HKR02).

Some participants stated in their reflection that activity experience and key competencies are also important dimensions of mathematics:

> Many years later, the complicated mathematical concepts and models we once learned, such as derivatives and conic curves, will be forgotten, but the way and ability to think acquired in the process are entrenched inside us, forming a neural memory that is of great significance for any future job or profession (YJY15).

To summarize, the PTs' BNM changed to some extent after experiencing the systematic program on PBL theories and practices. It is noteworthy that there was no significant change in the frequency (i.e., both 11) of PTs' use of the terms "foundational" or "emphasizing the basics" in the pre- and post-tests when describing their impressions of mathematics. Although the PTs were more agreeable and receptive to the problem-solving view, they still emphasized the importance of basic knowledge and skills in mathematics. This may be explained by the tradition of emphasizing the "double base" in teaching mathematics at the basic education level in China, as these participants had experienced when they were in primary and middle schools. Research has shown that this tradition may be a reason Chinese students perform well in international mathematics tests such as PISA (Guo et al., 2018). Therefore, it is positive that PTs maintain an emphasis on the "double-base" of mathematics before and after the course.

Knowledge organization: From "emphasizing knowledge logic" to "adapting to students' learning"

Terms such as "logical" and "emphasizing logic" were used more frequently in the pre-test to describe their impressions of mathematics (with a frequency of eight), while the usage of these terms decreased slightly in the post-test (with a frequency of five). Correspondingly, the usage of the terms "interesting" and "fun" differed significantly between the pre- and post-tests (with a frequency increase from six to 14). This suggests that PTs' BNM shifted from the Platonist view to the problem-solving view, with an increasing agreement that the organization of mathematical knowledge should be adapted to students' learning in addition to emphasizing the logic of knowledge.

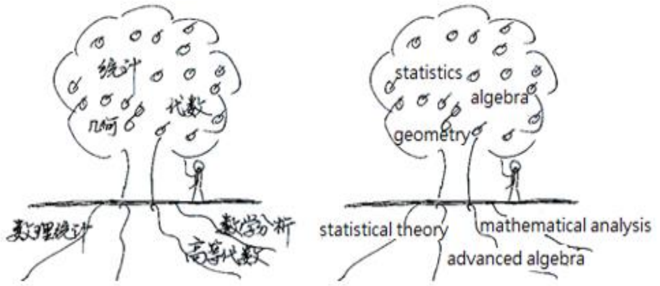

Figure 1. Mathematical imagery illustrated by participant YJY15 (the left and right are the original version in Chinese and the translated version, respectively) (Source: Authors' own elaboration)

For instance, a participant noted in the reflection:

> After experiencing the five PBL sessions, I increasingly feel and recognize the importance of the learning process and interest. Once upon a time, I thought that mathematical knowledge was deterministic and reasoned out by mathematicians, so it was the student's job to learn the standard answers. However, through this experience, I feel students seem to get an answer to a problem, but they are gaining a deeper logical ability by thinking about it and applying it to all aspects of their daily lives (CYB16).

In another instance, when describing "what mathematics is", participant MYY01 used the terms "interesting", "basic", "important", and "logical" in the pre-test but "basic", "logical", "variable", and "comprehension-based" in the post-test. Regarding "variable" and "comprehension-based", the participant explained in the post-test that,

> When using mathematical knowledge to solve real-world problems, it is necessary to combine abstract knowledge with concrete situations and to solve problems in unfamiliar situations with our familiar knowledge, so mathematics is variable. Middle school mathematics is comprehension-based, and students' comprehension of knowledge is a prerequisite for application. Only when students understand mathematical knowledge and know where it comes from and where it goes can they truly grasp its rationale and apply it to solve problems, which requires teachers to focus on students' level of understanding in teaching mathematics.

It can be seen that in addition to emphasizing the logic of knowledge, PTs also recognize its appropriateness for students' understanding. As advocated by Jingzhong Zhang of the Chinese Academy of Sciences, we need to move from a "mathematics education" orientation to an "educational mathematics" orientation when developing textbooks and designing instruction and to make necessary transformations of primitive, complicated mathematics so that it can be transformed from an academic form to an educational form and become learnable and teachable (Zhang & Cao, 2005). In his illustrated mathematical imagery (as shown in **Figure 1**), the participant, YJY15, explained it well with a funny limerick:

> The mathematical tree is always in bloom; I pick its fruit to nurture talent. The roots of the tree are intertwined in the mud that is invisible to us, but its meaning is as deep as the sea (the original version in Chinese is shùxué dà shù cháng shèngkāi, wǒ zhāi qí guǒ yù yīngcái. Pángēncuòjié nítǔ lǐ, wèi jiàn qí yì shēn sì hǎi).

In his mathematical imagery, mathematical analysis, advanced algebra, statistical theory, and other branches of higher mathematics constitute the logical system of mathematics, and for primary and middle school students, "algebra", "geometry" and "statistics" are the fruits that can be "picked" due to absorption of essence of the roots and the nourishment of the sun and rain.

Shifts in Beliefs About Learning Mathematics

From the responses to the pre-test, "student centered" was mentioned frequently in question 2. It can be seen that PTs have already agreed with this idea, so we will not elaborate on it here. In addition, most participants used such terms as "self-construction" and "teacher-student interaction" in the pre-test to describe their perceptions of mathematics learning, indicating that they already knew, understood, or accepted modern learning theories at the beginning of the course. These beliefs were reinforced by the systematic program on the theory and practice of PBL.

Learning approach: Increased agreement with "self-construction"

Participant ZJT05 mentioned in the pre-test that,

> Learning mathematics in middle school requires repeated practice to deepen understanding to build a foundation for learning more difficult content in high school. Mathematics education in compulsory education is not only about learning knowledge but also about leading students to discover new perspectives and paradigms for understanding the world, which is mathematical thinking and mathematical literacy.

In the post-test, this participant added that,

> Learning mathematics in middle school is progressively constructed, requiring both extensive, long-term practice and constant cognitive deepening so that it can spiral upwards.

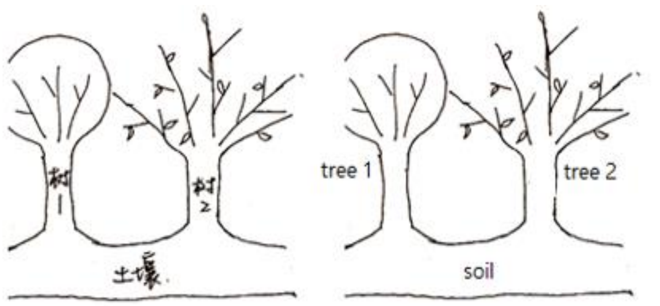

Figure 2. Mathematical imagery illustrated by participant XWJ06 (the left and right are the original version in Chinese and the translated version, respectively) (Source: Authors' own elaboration)

In the pre-test, participant QJQ08 recalled her own experience, stating,

> I have the impression that the atmosphere in the mathematics classes in middle school is very lively, and the teacher not only delivers knowledge but also often uses various teaching aids to help us better understand formulas and theorems, etc. The mathematics learning in middle school is in line with my cognitive and ability level.

The importance of personalized learning was further emphasized in the post-test:

> Mathematics learning should be tailored to the individual's needs, and teachers should be able to design the activities and teaching sequence in a reasonable way to achieve personalized learning.

Participant MXH09 focused on the progressive aspect of mathematics learning in the pre-test:

> Mathematics learning should form abstract thinking on the premise of having basic knowledge. For example, concepts such as triangles and parallelograms are already intuitively understood in students' life experiences, so mathematics learning should build on this foundation and move from intuitive perception to abstract understanding.

In the post-test, the participant further recognized the importance of self-construction in this process:

> Mathematical learning should emphasize individualized understanding. Learning should ideally happen in a relatively relaxed and democratic classroom environment, where students can develop the habit of proactive learning and take responsibility for their learning. In the process of learning, mathematical knowledge should be integrated with real life to achieve a unique understanding of the meaning of mathematical knowledge in life.

Similarly, participant FYR12 stated in post-test that,

> The mathematics classroom should guarantee students' opportunities for independent thinking and promote knowledge construction by integrating mathematical knowledge with students' personal experiences through the integration of real-life problems.

From the above, it can be seen that with the experience of the theory and practice of PBL, the PTs developed a deeper and clearer understanding of self-construction in learning mathematics. This is also reflected in the mathematical imagery illustrated by participant XWJ06 in the post-test (as shown in **Figure 2**):

> Each student has a tree of his or her own, which is rooted in the same "soil", namely, "mathematical knowledge", but each student has a unique approach to learning, so they all have a unique tree that is different from others. Because their understanding of knowledge is different, the tree grows in different ways.

Teacher-student relationship: Increased agreement with "interaction"

The participant LMX14 mentioned in the pre-test:

> How to learn mathematics in middle school? Just do more exercises, right? I just fell in love with the feeling of conquering one problem after another in middle school. My math teacher was a tall, thin, competent man. He drew diagrams quickly and could draw circles with his pinky finger as the center of the circle, without using a circle rule. So, we enjoyed listening to his geometric proofs, no matter how many times he drew them, we would all exclaim. He said that math is all about practicing more, so we were constantly given tests to complete on various topics, and later I came to love the sense of fulfilment in tackling problems. A classmate in my neighboring class often scored full marks in math, so I was most looking forward to getting full marks in math at that time.

The enjoyment of tackling problems seems to be the reason for this participant's love of mathematics, and the personal charm of the teacher drawing circles with bare fingers is also an important external factor. However, the participant added in the post-test that,

> Learning mathematics in middle school should be lively, relaxed, and intimate. I remember that my middle school math teacher had a close relationship with his students and was free to discuss difficult problems with him.

The importance of teacher-student interaction and classroom relationships was further emphasized.

In another instance, participant TQP11 stated in the pre-test that,

> Middle school mathematics can be mastered by doing more exercises. I have a solid foundation, so I can master the basic knowledge without much effort, and then I can keep improving my problem-solving skills through practice.

In the post-test, the participant reported that,

> The learning approach of middle school mathematics should be diversified and emphasize teacher-student interaction. Diversity means that there should be more emphasis on contextualized learning and that the classroom environment should not be dull. A particularly important point is that mathematics learning should shift from a teacher-centered approach to a student-centered approach.

Participant YZE04 had already recognized the importance of teacher-student interaction in the pre-test:

> I think the learning of middle school mathematics should be interactive and entertaining. In many cases, teachers do not need to focus on explaining all the time but rather allow students to explore and interact more on their own.

In the post-test, this participant stated more explicitly that,

> Learning in middle school mathematics should be teacher-student interactive, student–student interactive, and have a high degree of freedom. The teaching process should not be limited to lecturing and training but can be more activity-based, with an emphasis on cooperation and communication. In terms of the teacher-student relationship and student-student relationship, it should be possible to create opportunities for interaction so that they can inspire each other.

Similarly, the participant PSY23 stated the following in the post-test:

> In the middle school mathematics classroom environment, I think mutual communication is the most important, and both teacher-student interaction and student-student interaction are essential. Equality and respect are the cornerstones of teacher-student classroom communication, and good teacher ethics are the foundation on which this relationship is built.

It is worth noting that the teacher's role should not be neglected while emphasizing the "student-centered" approach; otherwise, the most freedom and relaxed environment will be reduced to false prosperity. In this regard, participant XWJ06 had already recognized in the pre-test that

> In terms of the teacher-student relationship, teachers should maintain a certain degree of dignity and seriousness in front of students. At the same time, teachers should also lower their position and give students more opportunities to demonstrate, and students' bursts of ideas should be considered and guided.

As another example, participant CYB16 pointed out in the post-test that

> The classroom environment should be group-based, with group members forming a circle where the teacher leads everyone to discuss and solve problems. In terms of the teacher-student relationship, I think teacher dignity should be necessary, but it is also important to establish a relationship with students that is both teacher and friend.

It is highly commendable that PTs could recognize the importance of the teacher's role while emphasizing student-student and teacher-student interactions.

Shifts in Beliefs About Teaching Mathematics

Teacher's role: From "knowledge interpreter" to "learning facilitator"

After experiencing the systematic program on the theory and practice of PBL, the PTs' perception of teachers' role changed gradually from the traditional perception of "knowledge interpreter" to "learning facilitator" to various extents. In her reflection, a participant noted that,

> Teachers should respect the centrality of students and stimulate their motivation. Mathematics teaching is not simply a process of teaching teachers and learning students. Since mathematical knowledge comes from life, teachers should impart realistic meaning to mathematical knowledge so that students can perceive mathematical knowledge from concrete contexts. Teaching should be conducted based on students' motivation, as the process is more important than the result (MYY01).

Other participants reflected that,

> Traditional mathematics teaching is teacher-centered and knowledge-oriented. An approach such as PBL shifts the center to the student, with more use of guidance, discovery, and other methods, where knowledge learning is more of a bottom-up construct and the process aspect of learning is valued (ZJT05).

Some participants suggested that teachers should not go to the extreme of pursuing knowledge teaching but should also promote students' learning by developing their abilities and stimulating their interests:

> I used to think that mathematics teaching is mainly about teaching knowledge, but now it seems that it is more important to teach students thinking skills and problem-solving abilities and, most importantly, to stimulate their interests. "Mathematics is fun" was once praised by mathematician Mr. Xingshen Chen. In the classroom of compulsory education, teachers should make more efforts to stimulate students' interest in learning and promote students' learning through their wisdom and art of teaching (LMX14).

> Mathematics teaching should focus on the richness of activities and mobilize students' interest and enthusiasm through diversified activities (CML17).

Teaching format: From "closed lecture" to "open inquiry"

The PTs' perceptions of teachers' roles have shifted, and these shifts are the prerequisite for changing their teaching behaviors. From the reflections, we find that the PTs' perceptions of teaching formats have changed gradually from "closed" to "open" and from "lecture" to "inquiry". For instance, participant HKR02 emphasized the importance of creating authentic contexts in teaching:

> Through the PBL experience, I found that mathematics teaching could be transformed from mechanical knowledge transfer to authentic context-based exploration. My previous perception of mathematics teaching was that the teacher delivers, and the students listen, but after this program, I found that teaching based on real-life situations makes it more productive. First, the students' interactivity is maximized when the teacher plays a guiding role. Second, context-based teaching makes students feel more realistic, and crafting project products provides students with motivation and a sense of accomplishment. Mathematics learning should be such an interactive exploration and rewarding process, not a passive acceptance.

For other examples,

> Mathematics teaching should first attract students' attention and interest, and it is better to start from real life and then return to real life (QJQ08).

> Teaching should not fall into the traditional misconception of filling a classroom with delivering but should pay attention to the connection between teaching content and real life so that students can experience "grounded" mathematics (CML17).

Some participants pointed out that,

> PBL transcends the traditional teaching of application problems but makes a great breakthrough in terms of task authenticity and sustained exploration: mathematics teaching should not be rigid but should give students more space for free inquiry (QSQ08).

> PBL escapes the limitations of application problems and emphasizes mathematical activities and practical tasks, which allows students the opportunity to explore and integrate knowledge into practice (YZE04).

Some considered that the cultivation of cooperative problem-solving skills could be realized in PBL:

> In PBL, students are confronted with problems that need to be solved, and through group work, they consult information, make plans, and complete tasks, and in this process, their skills are cultivated (XWJ06).

> I realized that teaching mathematics is not about making students learn word by word following textbook directions or repeatedly lecturing on exercises but about using activities such as class introduction, in-class activities, and group work to arouse students' interest and about making students learn to think independently and communicate cooperatively in the teaching process (HHZ20).

Other participants recognized the importance of mathematical communication:

> I used to think that because every question in mathematics has a standard answer and the teacher is the absolute authority, it is very normal for the teacher to lecture and the students to listen in the classroom. After a recent hands-on PBL design, I found that teacher-student communication is necessary, which is an important way to promote students' learning (CYB16).

> PBL is a good innovation in teaching that allows democracy and discussion in the classroom and establishes an open, inclusive, communicative, and cooperative learning environment (ZWB13).

Table 3. A holistic picture of the perceived shifts of the PTs' beliefs

	BNM		BLM		BTM	
Level	Objective orientation	Knowledge organization	Learning approach	Teacher-student relationship	Teachers' role	Teaching format
Pre-test	Concerned about "double-base"	Emphasizing knowledge logic	Agreement with "self-construction"	Agreement with "interaction"	Knowledge interpreter	Closed lecture
Post-test	Concerned about problem-solving	Adapting to students' learning	Increased agreement with "self-construction"	Increased agreement with "interaction"	Learning facilitator	Open inquiry
Shifts occurred	From instrumentalist view to problem-solving view	From Platonist view to problem-solving view	Increased agreement with problem-solving view	Increased agreement with problem-solving view	From instrumentalist view to problem-solving view	From instrumentalist view to problem-solving view

Summary of the Overall Shifts

From a holistic perspective, we illustrated the perceived shifts in PTs' beliefs in **Table 3**. An obvious finding was that most PTs' beliefs of BNM, BLM, and BTM reached the problem-solving level in the post-test, though they were at different levels in the pre-test. For example, BNM and BTM were at the level of instrumentalist view, while PTs' BLM was already showing some degree of problem-solving view in the pre-test, and then they most shifted from a lower level or strengthened by the original problem-solving view in the post-test. Therefore, through a systematic PBL theory and practice program, the PTs' beliefs of BNM were significantly shifted, which shift also positively influenced the shift in their BLM and BTM. In sum, the PBL program had positive effects on the perceived shifts of PTs' beliefs.

CONCLUSION AND DISCUSSION

The Experience of the PBL Theory and Practice Program Positively Influenced the Shift of Prospective Teachers' Beliefs

The current study found that after receiving a systematic PBL theory and practice program, the PTs' BNM, BLM, and BTM had shifted. We also confirmed previous research that shifts in PTs' beliefs usually begin with the BNM, and a shift in the BNM triggers a re-examination of their views on mathematics and teaching (Beswick, 2012; Cross, 2009). For instance, participant TQP11 expressed her perceptions of middle school mathematics in the pre-test as "rather boring", "full of mathematical symbols", "requires logical thinking", and "cold" but used the terms "fundamental", "interesting", "lifelike", and "applicable" in the post-test. It can be seen that her BNM shifted from the instrumentalist view to the problem-solving view. Correspondingly, her BLM and BTM also appeared to have changed in the post-test. For instance, in her reflection on the post-test, she noted that teaching should provide students with ample opportunities for engagement.

> In the past, I was perhaps limited by traditional models and methods and felt that mathematics teaching was mechanical. Through this period of PBL practice, I found that this approach to teaching math concerns the centrality of students and allows them to have maximum opportunities to be involved, thus making math teaching more meaningful.

This is supported by previous research showing that teachers' perceptions of mathematics influence their understanding of the design of instruction, and it is a prerequisite for shaping their BLM and BTM (Kutaka et al., 2018) and a factor that motivates them to transform their teaching behaviors (Rott, 2020; Thompson, 1992).

To summarize, shifting teachers' BNM is important for shifting their BLM and BTM. When teachers agree to view mathematical knowledge as a creation of human activity, they are more willing to devote more time, effort, and resources to encouraging student participation and exploration (Philippou & Christou, 2002). Our study further confirms that experiencing a systematic PBL theory and practice program can shift PTs' BNM, and thus their BLM and BTM, to a certain extent, when they are involved in the practice of PBL. When teachers are involved in the practice of PBL, they can feel a sense of achievement and fulfilment, and these tangible benefits eventually lead to a shift in their beliefs and a recognition and acceptance of this new approach to learning and teaching (Lavy & Shriki, 2008).

Programs on the Theory and Practice of Project-Based Learning Should be Incorporated into University Teacher Training Courses

Previous research has shown that challenges are frequently encountered in implementing PBL in primary and middle schools. The subjective reasons for these challenges are the teachers' concerns about the potential risks of this approach and their struggle to build trust in this emerging approach (Rogers et al., 2011). As curriculum implementers, teachers need a conceptual understanding and acceptance of PBL to be able to implement it accurately. The present study found that after an intervention with a program on the theory and practice of PBL, prospective beliefs shifted to some degree. The PTs thought the short program was beneficial for their professional development:

> Through this program, I was exposed to many projects that could spark students' interest in teaching middle school mathematics, and I also saw new ideas and practices. Although my understanding of PBL is not yet thorough, I have already experienced a pedagogical innovation in the design process, which also has a high value for future professional development (YJY15).

This suggests that before implementing PBL as an emerging approach on a large scale in middle schools, we should first help teachers understand and embrace its ideas. Moreover, our attempts have proven that it is necessary, feasible, and somewhat productive to train them at both theoretical and practical levels.

The intervention in this study was designed for PTs with a relatively short program period. Although the participants showed some degree of change in their beliefs, we suggest that it is still necessary to be relatively conservative about what can be accomplished. Shifting teachers' beliefs is not easy. A previous study on in-service teachers showed that after a year of experimenting with PBL, some teachers' perspectives and approaches to mathematics teaching remained virtually unchanged (Rogers et al., 2011). Other studies have noted that to properly understand PBL as described in the document, teachers need to receive long-term training and years of practice (Markula & Aksela, 2022; Mentzer et al., 2017). Studies have even found that although teachers form new beliefs, this does not necessarily mean that they have abandoned their old beliefs (Ambrose, 2004; Bernack-Schüler et al., 2015). This was confirmed in the present study, as participant TQP11 mentioned in her reflection:

> The design of PBL requires high requirements for teachers, not only a strong sense of innovation but also a continuous framing and refinement by the teacher team, which is a long process of exploration.

Therefore, in the context of PBL increasingly becoming a dominant teaching approach in middle schools, designing systematic training programs on PBL with more content, better structure, and longer duration is crucial for both prospective and in-service teachers to shift their beliefs and realize the ideas of PBL. In this regard, further research has yet to be conducted. Based on the short-term training of PTs in this study, the following two suggestions could be considered. First, we design an intuitive theoretical curriculum. For frontier teachers, abstract theoretical knowledge needs to be presented in an easy-to-understand and operational way. In this regard, "6A standards" and "Gold standards" developed by Buck Institute for Education are recommended (see https://www.pblworks.org/). Second, teachers should be encouraged to be involved in the design and implementation of PBL and to reflect on the experience. Research has shown that beliefs can influence actions, while experiences and reflections can also influence beliefs (Lavy & Shriki, 2008). When confronted with unprecedented ideas and challenges, teachers' deeply held beliefs may be shocked and thus shifted (Lloyd, 2002). This was also evidenced in this study.

Overall, by providing PTs with a systematic theoretical and practical program on PBL in the teacher training curriculum, this study found that such program experience is feasible for shifting PTs' beliefs. Specifically, three main findings were derived from this study. First, a systematic theoretical and practical program on PBL resulted in observable shifts in all three dimensions of PTs' beliefs. Second, the shifts in BNM were mainly in the perceptions of objective orientation and knowledge organization, the shifts in BLM were mainly in the perceptions of learning approaches and teacher-student relationships, and the shifts in BTM were mainly in the perceptions of teacher roles and teaching formats. Third, the shift in beliefs usually begins with BNM (Baron, 2015), so it is in the dominant position in the belief system of PTs.

Limitations and Further Consideration

The findings are not representative or generalizable due to the small sample size. However, we consider that teachers' beliefs are a complicated and latent concept and that the method applied is probably a better way to measure them by teachers' articulating their inner thoughts. Interestingly, in a previous study (Rogers et al., 2011), two of three in-service teachers were found to not have shifted their beliefs significantly after a year of PBL practice. The conclusions of this study seem to be more positive. One possible explanation may be that the PTs had not yet developed their teaching style and teaching philosophy, and therefore, their beliefs were more sensitive to new input than those of the in-service teachers. However, we consider at least two other reasons for the current results. Reflections on these reasons may inform future research.

First, although considerable research has supported the idea that teachers' beliefs impact their teaching practices (Ball & Cohen, 1996; Cross, 2009; Philipp, 2007; Torff, 2005; Wilkins, 2008; Wilson & Cooney, 2002; Lavy & Shriki, 2008), their behavior in the classroom may be inconsistent with their beliefs (Chen, 2008; Vesga-Bravo et al., 2022), and reported beliefs may differ from their actual beliefs (i.e., those reflected in daily teaching behaviors) (Beswick, 2012; Speer, 2005). It has also been noted that one of the flaws in research on teachers' beliefs is the attempt to explore the structure of beliefs through an empiricist approach that assumes that teachers can accurately express their beliefs (Leatham, 2006). This implies that to truly capture teachers' beliefs, valid information needs to be inferred from their practices, which include but are not limited to their daily teaching behaviors, the design of the teaching process, and their perceptions of treating students. Regarding a better understanding of PTs' beliefs, a possible way might be to judge them through their designed instructional planning documents.

Second, as suggested by existing research, teachers' perceptions of their belief shifts may be exaggerated, and sometimes teachers self-report significant shifts while experiencing limited shifts (Rogers et al., 2011). With this in mind, although the majority of participants in the present study reported positive shifts in beliefs, we need to be conservative about interpreting this result. Whether the beliefs changed still needs to be analyzed in his or her performance in teaching practice.

Third, for comparison purposes, the questionnaire should be the same in the pre- and post-test. In the current study, we did not include the question "what do you know about teaching mathematics in middle school" in the pre-test with consideration of the PTs' limited teaching experience. They may still have their beliefs in teaching before the course, which could have come from their experience or interaction with their teachers.

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